In cases where the sample is small, the distribution is unknown and the evidence seems to point very strongly against the null hypothesis, we may use Chebyshev's inequality to estimate the P-value, as in the next example.

Example 7.2.7 (Age of First Marriage in Ancient Rome). Lelis, Percy and Verstraete[5] studied the ages of Roman historical figures at the time of their first marriage. They did this to refute earlier improbably high age estimates that were based on funerary inscriptions. Others had found that for women, the epitaphs were written by their fathers up to an average age of 19 and after that by their husbands, and jumped to the conclusion that women first married at an average age of 19. (A similar estimate of 26 was obtained for men.)

From the historical record, the ages at first marriage of 26 women were 11, 12, 12, 12, 12, 13, 13, 13, 13, 13, 14, 14, 14, 14, 14, 14, 15, 15, 15, 15, 15, 15, 16, 16, 17, 17.

The mean of these numbers is 14.0 and the standard deviation is 1.57.

A random sample of size 26, is just barely large enough to assume that the average is normally distributed with standard deviation $1.57/\sqrt{26} \approx 0.31$, nevertheless, we first assume this, but then obtain another estimate without this assumption as well.

This sample, however, is a sample of convenience. We may assume though that it is close to a random sample, at least from the population of upper class women. We also assume that marriage customs remained steady during the centuries covered. (For this reason, we omitted three women for whom records were available from the Christian era.)

We take the null hypothesis to be that the average is 19, and the alternative hypothesis to be that it is less. With the above assumptions, we can compute the P-value, that is, the probability that the mean in the sample turns out to be 14 or less if the population mean is 19, as

$$\mathrm{P}(\overline{X} \leq 14) = \mathrm{P}\left(\frac{\overline{X} - 19}{0.31} \leq \frac{14 - 19}{0.31}\right) \approx \Phi\left(\frac{14 - 19}{0.31}\right) \approx \Phi(-16) \approx 0.$$

Thus, the null hypothesis must be rejected with practical certainty, unless the assumptions can be shown to be invalid.

The ridiculously low number we obtained, depends heavily on the validity of the normal approximation, which is questionable. We can avoid it and compute an estimate for the P-value by using Chebyshev's inequality (see Theorem 5.2.6) instead, which is valid for any distribution. Using the latter, we have $\mathrm{P}(|\overline{X}_n - \mu| > \varepsilon) = \mathrm{P}(|\overline{X}_n - 19| > 5) \leq \sigma^2/n\varepsilon^2 \approx 1.57^2/26 \cdot 5^2 \approx 3.8 \times 10^{-3}$. This estimate, though very crude (in the sense that the true P-value is probably much lower), is much more reliable than the one above, and it is still sufficiently small to enable us to conclude that the null hypothesis, of an average age 19 at first marriage, is untenable.

[5] A. A. Lelis, W. A. Percy and B. C. Verstraete, *The Age of Marriage in Ancient Rome* (The Edwin Mellen Press, 2003)